

# 1 Independence

In class, I said that if events  $E$  and  $F$  are independent, then

- $E$  and  $F'$  are independent;
- $E'$  and  $F$  are independent; and,
- $E'$  and  $F'$  are independent.

First, let us see why  $E$  and  $F'$  are independent if  $E$  and  $F$  are. Notice that  $E = (E \cap F) \cup (E \cap F')$  (if you are not sure why this is true, draw a Venn diagram!). Also, notice that  $(E \cap F)$  and  $(E \cap F')$  are disjoint. So

$$\Pr[E] = \Pr[E \cap F] + \Pr[E \cap F'].$$

Now, since  $E$  and  $F$  are independent, we have  $\Pr[E \cap F] = \Pr[E] \Pr[F]$ . So, the above equation can be written as

$$\Pr[E] = \Pr[E] \Pr[F] + \Pr[E \cap F'].$$

Rearranging the terms, we see

$$\Pr[E \cap F'] = \Pr[E] - \Pr[E] \Pr[F] = \Pr[E](1 - \Pr[F]) = \Pr[E] \Pr[F'].$$

That's why  $E$  and  $F'$  are independent if  $E$  and  $F$  are.

Using exactly the same argument, we can show that  $E'$  and  $F$  are independent if  $E$  and  $F$  are.

You can also show the third bullet point using similar argument. But if you understand the statements well, you will see that it actually follows directly from the first two bullet points.